

# Forms of Mathematization

Sophie Roux\*

Université de Grenoble / Institut universitaire de France

According to a grand narrative that long ago ceased to be told, there was a seventeenth century Scientific Revolution, during which a few heroes conquered nature thanks to mathematics.<sup>1</sup> This grand narrative began with the exhibition of quantitative laws that these heroes, Galileo and Newton for example, had disclosed: the law of falling bodies, according to which the spaces traversed by a falling body are to each other as the squares of the times in which they are traversed; the law of gravitation, according to which two bodies are attracted to one another in proportion to the product of their masses and in inverse proportion to the square of the distance separating them — according to his own preferences, each narrator added one or two quantitative laws of this kind. The essential feature was not so much the examples that were chosen, but, rather, the more or less explicit theses that accompanied them. First, mathematization would be taken as the criterion for distinguishing between a qualitative Aristotelian philosophy and the new quantitative physics. Secondly, mathematization was founded on the metaphysical conviction that the world was created in *pondere, numero et mensura*, or that the ultimate components of natural things are triangles, circles, and other geometrical objects. This metaphysical conviction had two immediate consequences: that all the phenomena of nature can be in principle submitted to mathematics

---

\* PLC, BP 47, 1281 avenue Centrale 38040, Grenoble Cedex 9, France (Sophie.Roux@upmf-grenoble.fr). Except for one, the essays presented here were originally delivered as talks during the seminar *La mathématisation comme problème* that was organized at the university of Grenoble between January 2007 and June 2008 on a monthly basis, and thereafter discussed during an editorial meeting that took place in Lyon in January 2009. *Cluster 14* of the Région Rhône-Alpes provided the financial help that these academic events required. Edith Sylla generously agreed to contribute to this fascicle without having participated to the seminar and to read an earlier version of this introduction, and I thank her for that. I wish also to thank all participants to the seminar for the lively and sometimes heated discussions we had, the editor of *Early Science and Medicine* for his willingness to consider the essays for a special issue, some anonymous referees for their sharp comments and, finally, Jürgen Renn for having invited me to the Max Planck Institut für Wissenschaftsgeschichte throughout the period when this fascicle was being completed. During my stay at the MPIWG, I met Fokko Jan Dijksterhuis, who is currently working on a similar project, *The Uses of Mathematics in the Dutch Republic*: I regret that we did not meet earlier.

<sup>1</sup> Edmund Husserl and Alexandre Koyré are the two key-figures in the emergence of the grand narrative of mathematization of nature. Husserl claimed that Galileo was the first to mathematize nature, i.e., according to Husserl, to surreptitiously substitute mathematical idealities for the concrete things of the intuitively given surrounding world (*The Crisis of European Sciences and Transcendental Phenomenology. An Introduction to phenomenological philosophy*, Engl. tr. David Carr (Evanston, IL, 1970), § 9, 23-59). Koyré introduced Husserlianism in the history of science and argued that the mathematization (idealization) of nature triggered the Scientific Revolution from Galileo to Newton. For a first presentation of Koyré's theses, see Gérard Jorland, *La science dans la philosophie. Les recherches épistémologiques d'Alexandre Koyré* (Paris, 1981); on the influence that § 9 of the *Crisis* may have had on Koyré, see François De Gandt, *Husserl et Galilée. Sur la crise des sciences européennes* (Paris, 2005), 97-103. Mathematization of nature is assumed to epitomize the Scientific Revolution in the very titles of the following classical studies, whether or not they endorse idealism: William Shea (ed.), *Nature Mathematized. Historical and Philosophical Case Studies in Classical Modern Natural Philosophy* (Dordrecht, 1983); Joella G. Yoder, *Unrolling Time: Christiaan Huygens and the Mathematization of Nature* (Cambridge, 1988); Lino Conti, *La matematizzazione dell'universo: momenti della cultura matematica tra '500 e '600* (S. Maria degli Angeli, 1992); Michel Blay, "La mathématisation de la nature," in *L'Europe des sciences. Constitution d'un espace scientifique*, ed. Michel Blay and Efthymios Nicolaidis (Paris, 2001), 115-134, *passim*. In the following footnotes, I shall give a few non-historical references, in order to make clear that idealism is not the only relevant philosophical tradition.

and that mathematical language is transparent, since it is the language of nature itself. Finally, it goes without saying that, from a social point of view, the evolution of the sciences was apprehended through what has been aptly called the “relay runner model,” according to which science progresses as a result of individual discoveries.<sup>2</sup>

Grand narratives such as this are perhaps simply fictions doomed to ruin as soon as they are clearly expressed. In any case, the very assumption on which this grand narrative relies can be brought into question: even in the domain of mechanics, that was supposedly crucial to understand the Scientific Revolution, the relevant epistemological units crucial to understanding the dynamics of the Scientific Revolution are perhaps not a few laws of motion, but a complex set of problems embodied in mundane objects.<sup>3</sup> Moreover, each of the theses just mentioned was actually challenged during the long period of historiographical reappraisal, out of which we have probably not yet emerged.

Against the sharp distinction between a qualitative Aristotelian philosophy and the new quantitative physics, numerous studies insist that Rome wasn’t built in a day, so to speak. Since Antiquity, there have always been mixed sciences; the emergence of pre-classical mechanics depends on both medieval treatises and the practical challenges met by Renaissance engineers. It is indeed true that, for Aristotle, mathematics merely captures the superficial properties of things, but the Aristotelianisms were many during the Renaissance and the early modern period, with some of them being compatible with the introduction of mathematics in natural philosophy.<sup>4</sup> In addition, the gap between the alleged program of mathematizing nature and its effective realization was underlined: as most natural phenomena actually escaped mathematization, at best they were enrolled in what Thomas Kuhn began to rehabilitate under the appellation of the “Baconian sciences,” i.e., investigations aiming at establishing isolated facts or empirical generalizations, without relating them to any theory, whether mathematical or not.<sup>5</sup> Hence, mathematization of nature cannot pretend to capture a historical moment: at most, it expresses an indeterminate task for generations to come.

On top of these first two considerations, and against the thesis of the neutrality of the mathematical language, it was urged that mathematics is not “only a language” and that, exactly as other symbolic means or cognitive tools, it has its own constraints.<sup>6</sup> For example, it has been thoroughly explained that the Euclidean theory of proportions both guides and frustrates the Galilean analysis of motion; its shortcomings were particularly clear with respect to the expression of continuity, which is crucial in the case of motion.<sup>7</sup> Consequently, when calculus was invented and applied to the analysis of motion, it was not a transposition that left things as they stood. Even more clearly than in the case of a translation from one

---

<sup>2</sup> Jochen Büttner, Peter Damerow, Jürgen Renn, “Galileo’s Unpublished Treatises. A Case Study on the Role of Shared Knowledge in the Emergence and Dissemination of an Early Modern ‘New Science’,” in *The Reception of the Galilean Science of Motion in Seventeenth-Century Europe*, ed. Carla Rita Palmerino and J.M.M.H. Thijssen (Dordrecht, 2004), 99–117, here 100.

<sup>3</sup> Alan Gabbey, “The Case of Mechanics: One Revolution or Many?” in *Reappraisals of the Scientific Revolution*, ed. David C. Lindberg and Robert S. Westman (Cambridge, 1990), 493-528; Domenico Bertoloni Meli, *Thinking with Objects. The Transformation of Mechanics in the Seventeenth Century* (Baltimore, 2006), 1-6, *passim*.

<sup>4</sup> Peter Damerow, Gideon Freudenthal, Peter McLaughlin and Jürgen Renn, *Exploring the Limits of Preclassical Mechanics* (2<sup>nd</sup> ed., New York, 2004); Walter R. Laird and Sophie Roux (ed.), *Mechanics and Natural Philosophy before the Scientific Revolution* (Dordrecht, 2008), *passim*.

<sup>5</sup> Thomas S. Kuhn, “Mathematical versus Experimental Tradition in the Development of Physical Science,” in *Idem, The Essential Tension* (Chicago, 1977), 31-65, here 41-51.

<sup>6</sup> Gaston Bachelard, *L’activité rationaliste de la physique contemporaine* (Paris, 1951), 28-29; Gilles-Gaston Granger, *Essai d’une philosophie du style* (Paris, 1969), 21-24.

<sup>7</sup> Enrico Giusti, “Il filosofo geometra. Matematica e filosofia naturale in Galileo,” *Nuncius*, 9 (1994), 485-498; *id.*, “Galileo e le leggi del moto,” in Galileo Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, ed. E. Giusti (Turin, 1990).

natural language to another, the shift from one symbolic language to another entails that certain possibilities are opened while others are closed.<sup>8</sup> The cognitive constraints imposed by established mathematical theories, as seen in the theory of proportions or calculus, were not the only ones to be studied in relation to mathematization. Certain schemes dependent on the grammar of natural languages, e.g., the scheme of contrariety, or certain symbolic means of representation, e.g. geometrical diagrams and numerical tables, were also subject to such scrutiny.<sup>9</sup>

Lastly, it was insisted that, even if we concede the existence of scientific geniuses, mathematics is largely produced by intellectual communities and embedded within social practices. More attention was consequently paid to the forms of communication in given mathematical networks, or to the teaching of the discipline in, for example, Jesuit colleges and universities.<sup>10</sup> The set of mathematical practices specific to specialized craftsmen, highly-qualified experts and engineers began to be studied in its own right.<sup>11</sup>

All these reflections might have helped us change our perspectives on the question of mathematization. It seems, however, that they were instead set aside, both because of a general distrust towards sweeping narratives that are always subject to the suspicion that they overlook the unyielding complexity of real history, and because of a shift in our interests. The more obscure and idiosyncratic they are, the more an alchemist, a patron of the sciences or a lunatic collector is nowadays honored in journals of the history of sciences. As for the general issues involved in the question of mathematization, they are rejected as obsolete, or reserved for specialized journals in the history of mathematics. Consequently, before presenting the essays of this fascicle, I would like to say a few words in favor of a renewed study of the forms of mathematization in the history of the early sciences.

In general, the term “mathematization” refers to the application of concepts, procedures and methods developed in mathematics to the objects of other disciplines or at least of other fields of knowledge. A definition of this kind seems to assume that there is an agreement, first, on what mathematics is, second, on the profits that various disciplines can make out of its application and, third, on the relevance of the very notion of application. But there are many good reasons to think that such an agreement might be difficult to achieve.

There was never a working definition of mathematics in general; even at the time when the traditional definition of mathematics as the “science of quantities” or “magnitudes in general” emerged and was commonly accepted, there were different conceptions of

---

<sup>8</sup> Michel Blay, *La naissance de la mécanique analytique : la science du mouvement au tournant des XVII<sup>e</sup> et XVIII<sup>e</sup> siècles* (Paris, 1992); *id.*, *Les raisons de l’infini: Du monde clos à l’univers mathématique* (Paris, 1993), Engl. tr. by M. B. DeBevoise, *Reasoning with the Infinite. From the Closed World to the Mathematical Universe* (Chicago, 1998).

<sup>9</sup> Peter Damerow and al., *Exploring the Limits*, 71-134. For diagrams, see, moreover, the references given below, footnotes 25 and 26; for numerical tables, see Bertoloni Meli, *Thinking with Objects*, p. 109-112, p. 131-133, 179-181, 226-227; *idem*, “The Role of Numerical Tables in Galileo and Mersenne,” *Perspectives on Science*, 12.2 (2004), 164-190.

<sup>10</sup> For communication of mathematics, see Jeanne Peiffer, “Faire des mathématiques par lettres,” *Revue d’histoire des mathématiques*, 4 (1998), 143-157; *ead.*, “Communicating Mathematics in the Late 17<sup>th</sup> Century. The Florentine Cupola,” in *The Circulation of News and Knowledge in Intersecting Networks*, ed. Sven Dupré and Sachiko Kusukawa (Oxford, 2008), 92-119; Catherine Goldstein, “L’arithmétique de Fermat dans le contexte de la correspondance de Mersenne : une approche micro-sociale,” *Annales de la Faculté des sciences de Toulouse*, 18 (2009), 25-57. For teaching of mathematics, see Moredechai Feingold, *The Mathematicians’ Apprenticeship: Science, Universities and Society in England (1561-1640)* (Cambridge, 1984); Antonella Romano, *La Contre-réforme mathématique : constitution et diffusion d’une culture mathématique jésuite à la renaissance (1540-1640)* (Rome, 1999).

<sup>11</sup> For the use of mathematics by engineers, see Hélène Vérin, *La gloire des ingénieurs : l’intelligence technique du XVI<sup>e</sup> au XVIII<sup>e</sup> siècle* (Paris, 1993), 167-187, 281-293, *passim*. On the rationalization of technical practices in general, see Pacal Dubourg Glatigny and Hélène Vérin (eds.), *Réduire en art. La technologie de la Renaissance aux Lumières* (Paris, 2008). See moreover the references given below, footnote 21.

quantities, and consequently different ways of conceiving of the unity of mathematics.<sup>12</sup> Now, if the second-order question of how to define mathematics has been raised, it is because it is a fundamentally complex field, that has included various domains from its very beginning and that has kept developing new domains throughout history.<sup>13</sup> Even pure mathematics underwent many changes between the fourteenth and the seventeenth centuries. The reappropriation of ancient texts was crucial in the Renaissance. The point was to reconstruct the methods and the results that ancient mathematicians, such as Archimedes, Apollonius or Diophantus, had supposedly discovered. In the seventeenth century, new objects were examined (for example, new curves like the cycloid or the catenary); ancient problems were solved through new methods (here one can think of Cavalieri's geometry of indivisibles or of the Cartesian algebraization of geometry); new methodological demands were formulated (the exclusion of mechanical curves from Descartes' geometry or the devaluing of proofs by superposition or contradiction); and, last, with the emergence of calculus and of infinite series, new domains began to be explored.<sup>14</sup> In these circumstances, we should neither look for a definition of mathematics in general, nor think of mathematics as a unified field of knowledge, but, rather, submit to an historically situated and empirical definition of mathematics, namely what should be called "mathematics" is the activities of those who called themselves or were called by others "mathematicians." As tautological and circular as it may appear, such a definition is not without consequence for how we should conceive of mathematization.

Still confining ourselves to pure mathematics, if mathematics itself is diverse, it comes as no surprise that it gives rise to different kinds of mathematization, each of which brings its own benefits. Arithmetic, in as far as it is the practice of numbers, generates a first form of mathematization: then "quantification" consists in capturing in numerical form certain aspects of material things. Such a capture requires not only measurements, concrete apparatus and a concern for precise and standardized data, but also graphical techniques to present numerical results and intellectual techniques of approximation and averaging. Of course, quantification may be only peripherally related to the disinterested search of laws in natural philosophy: the alleged benefits of quantification are sometimes practical. As for geometry, it is implied not only in the highly codified practices of demonstration that involve lettered diagrams and ritual formulas, but more generally in the proto-mathematical practices of spatial representation that appear in treatises theorizing procedures of surveying, the art of cartography, the use of linear perspective in painting, etc. Hence, "geometrization" has a wide range of meanings, but its specific benefits are in general related to those of spatial visualization. If arithmetic and geometry were pregnant in the early sciences, the new symbolic algebra raised an additional hope. Insofar as it is a blind and procedural manipulation of signs, it was seen as leading to a universal science that would be applicable to anything without regard to particulars. Now, this multiplicity does not come only from the diversity of mathematical fields: mathematical practices themselves have different facets, and each of these different facets may suggest a certain kind of mathematization. For example, the apodictic way in which mathematics conducts its demonstrations and the necessity it confers on its results exerted an enduring fascination. Remote as it may be from the actual practices of mathematicians, the ensuing demand for *more geometrico* presentations implies significant

---

<sup>12</sup> This is shown in relationship with the Aristotelian prohibition rule on kind-crossing by Paola Cantu, "Aristotle's Prohibition Rule on Kind-crossing and the Definition of Mathematics as a Science of Quantities," *Synthese*, 174 (2010), 225-235.

<sup>13</sup> Even though it concerns Antiquity, I find Serafina Cuomo, *Ancient Mathematics* (London, New York, 2001), very helpful here.

<sup>14</sup> Besides the works mentioned footnotes 8, 24 and 29, see Niccolò Guicciardini, *The Development of Newtonian Calculus in Britain* (Cambridge, 1989); Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (New York, Oxford, 1996).

constraints. We can speak of “axiomatisation” in this case, even when the identification of a coherent set of axioms and definitions, from which to derive the other propositions, was not completely operative.

Calling “mathematics” the activities of those who called themselves, or were called by others, “mathematicians” has a second consequence: we should stop thinking of mathematization as an application of pure mathematics. In the early sciences, what was contrasted was not “pure” versus “applied” mathematics, but “pure or abstract” versus “mixed” mathematics, and “speculative or theoretical” versus “practical” mathematics.<sup>15</sup> The problem is not just a question of words. The nineteenth century notion of application and the correlative opposition between “pure mathematics” and “applied mathematics” are based on the idea that there are certain mathematical procedures that do not need to be transformed to give satisfying explanations of the phenomena under consideration.<sup>16</sup> This idea is indeed adapted to certain cases: even if a ray of light is not a straight line, the geometry of the straight line was applied to the ray of light; even if the theory of proportions was not formulated for the analysis of motion, it was applied to the analysis of motion. But, as we have seen, early modern mechanics shows that the analysis of certain phenomena may require not only a transformation of the existing mathematical tools, but, ultimately, the constitution of new mathematical objects. Moreover, the notion of application does not seem appropriate for arts that were qualified as “practical mathematics” in the sixteenth and seventeenth centuries: surveying, building fortifications and canals, navigation, architecture, book-keeping, inventing machines for war and/or peace, drawing perspective, etc. Mathematics was indeed mobilized in these practices, but since the mathematics of the time was not able to explain them satisfactorily, the contemporary notion of application does not help to see what is at stake in them. We might, rather, understand these practices as proto-mathematical material that formal mathematics theorizes. The proto-mathematical (for example, land-surveying) is historically prior to the mathematical (in this case geometry); the proto-mathematical material derives from a human symbolical practice (triangles are not found in nature, but constructed according to definite procedures by humans that have social needs and symbolical means). Mathematics consists of inserting this proto-mathematical material into a second-order discourse.<sup>17</sup>

These remarks may lead to the conclusion that the grand narrative about mathematization of nature has to be enriched with the dense spectrum of various

---

<sup>15</sup> For the history of these distinctions, see Richard D. McKirahan Jr., “Aristotle’s Subordinate Sciences,” *British Journal for the History of Science*, 11 (1978), 197-220; James G. Lennox, “Aristotle, Galileo, and ‘Mixed Sciences,’” in *Reinterpreting Galileo* (Washington, D.C., 1986), ed. W. Wallace, 29-51; Stephen J. Livesey, “William of Ockham, the Subaltern Sciences, and Aristotle’s Prohibition of Meta-basis,” *British Journal for the History of Science*, 18 (1985), 127-145; Gary I. Brown, “The Evolution of the Term ‘Mixed Mathematics,’” *Journal of the History of Ideas*, 52 (1991), 81-102; Jean-Marc Mandosio, “Entre mathématiques et physique : Note sur les ‘sciences intermédiaires’ à la Renaissance”, in *Comprendre et maîtriser la nature au Moyen Age: Mélanges d’histoire des sciences offerts à Guy Beaujouan* (Geneva, 1994), 115-138; Walter Roy Laird, “Galileo and the Mixed Sciences,” in *Method and Order in the Renaissance Philosophy of Nature*, ed. Daniel A. Di Liscia, Eckhard Kessler and Charlotte Methuen (Aldershot, 1997), 253-270.

<sup>16</sup> These characteristics of the notion of application were first identified by Louis Althusser, *Philosophie et philosophie spontanée des savants* (Paris, 1967), 31-32, who, moreover, claimed that the relation of mathematics to physics, is not a relation of “application,” but a relation of “constitution,” in as far as mathematical languages “constitute” the objects of physical mathematics. In French scholarship at least, Althusser’s point was popularized by Jean-Marc Lévy-Leblond, “Physique et mathématique,” in *Penser les mathématiques* (Paris, 1982), 195-210. There was consequently a tendency to privilege “true mathematization” over “simple application,” but the more I think about it, the more I think that the conceptual distinction is justified, but that the devaluation of “simple application” is an historiographical dead-end.

<sup>17</sup> Paul Raymond, *Le passage au matérialisme* (Paris, 1973), 329-331. Enrico Giusti, *Ipotesi sulla natura degli oggetti matematici* (Turin, 1999) helps to clarify this hypothesis and to show that it may be adapted to more complicated and better documented cases than the hypothetical emergence of geometry from land-surveying.

mathematical practices. And, indeed, leaving behind the idealities that Husserl and Koyré waved at and replacing them with real practices such as manipulating numbers, extracting roots, representing perspective in pictures, compounding proportions, arranging numbers in tables, following rules and algorithmic procedures, linking propositions together, visualizing magnitudes in geometric diagrams, solving problems, measuring fields with specific instruments, drawing curves, making deductions and plotting the routes of ships, was a significant and much needed change of scenery.<sup>18</sup> However, the term “practice” seems today to be so indefinitely extendable, that it is not easy, beyond the slogan, to know exactly what it means. This ambiguity has not been without intellectual benefits, since it helped us to come out of idealism, but we must now ask ourselves what speaking of practices commits us to.

It seems to me that the term “practices” has at least three accepted usages, which are not mutually exclusive, but which should help us in understanding where we stand. First, to speak of mathematical practices may imply that a descriptive and historical point of view is adopted, rather than a normative and philosophical perspective; in that sense, the term “practice” refers simply to “mathematics as it is done, not as it should be done according to some preconceived philosophical viewpoint.”<sup>19</sup> As the previous pages suggest, adopting such a point of view ineluctably leads one to take into account a swarm of small problems and local methods, which generate an image of mathematics quite different from the still received view, according to which mathematics is expected to grow continuously and uniformly from a limited set of definitions and axioms. Second, practices may refer to the non-verbal commitments shared by mathematicians which help to define a scientific style and to constitute an intellectual community. Practices in this sense are opposed to explicit beliefs and may be invisible to the mathematicians themselves. Here I am thinking of a book — which may be idiosyncratic in this respect — that is concerned with cognitive practices that may be interpreted as logical conditions for the possibility of the emergence of the Greek deductive style.<sup>20</sup> Third, in accordance with the actors’ categories, mathematical practices can be identified with practical mathematics, as contrasted with pure mathematics and can refer to the real world, with its economic interests, practical concerns, material instrumentation, local settings and complex social networks. Even Kuhn’s rehabilitation of the Baconian sciences relied on a clear-cut separation between the mathematical tradition and the experimental tradition, and also on a high valuation of the abstract realm of pure ideas to the detriment of the concrete world of allegedly confused practices. The historiographical function of such references to mathematical practices at this point is neither more nor less than bridging the gap between intellectual and social history.<sup>21</sup>

The essays of this fascicle are by and large concerned with practices in the first sense: their point is neither to identify the formal conditions of possibility of deduction, nor to follow the practical engagements of scientists in the real world out there, but to capture some aspects of mathematics as it was pursued in various fields of knowledge. With this in mind, the title *Forms of Mathematization* was chosen for this fascicle. But, of course, forms are not Platonic ideas here: mathematical objects are not given once and for all in an ideal world, they are historically constructed. The notions of operative knowledge and of knowing by

---

<sup>18</sup> Note in this respect that neither Husserl nor Koyré paid attention to such practices, not even to the practices of pure mathematics.

<sup>19</sup> Mancosu, *Philosophy of Mathematics*, 4.

<sup>20</sup> Reviel Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History* (Cambridge, 1999), 3-7.

<sup>21</sup> Jim Bennett, “The Challenge of Practical Mathematics,” in *Science, Culture and Popular Belief in Renaissance Europe*, ed. Stephen Pumfrey and al. (Manchester, 1991), 176-190; *id.*, “Practical Geometry and Operative Knowledge,” *Configurations* 6.2 (1998), 195-222. The blurring of Kuhn’s two traditions is also manifest in the ambiguous nature of the “objects” studied by Bertoloni Meli, *Thinking with Objects*, see in particular 1-6, 310-316.

doing should be taken seriously when we deal with pure mathematics, because in order to know-that (this is true), mathematicians begin by knowing-how (to draw a diagram, to find the solution of an equation, to construct a curve, to make an operation, etc.).<sup>22</sup>

The three first essays of this fascicle, by Sylla, Boulier and Palmerino, are devoted to an classic question as far as mathematization of nature is concerned; namely, the question of mathematization of motion. These essays do not, however, aim at identifying the roads that led to the discovery of one law of motion or another. Rather, they bring to the fore certain specific forms of mathematization that were associated with the mathematization of motion: deductive reasonings on motion, alternative mathematical methods for the analysis of motion and geometrical diagrams representing motions.

Sylla focuses on the community of the fourteenth century Oxford Calculators (William Heytesbury, John Dumbleton and Richard Swineshead). The first challenge of her paper is to cast light on the kind of mathematization involved in the formulation of the middle-degree theorem without applying the prism of Oresme — who introduced the first graphic representation of this theorem — nor seeing this theorem through Galilean glasses.<sup>23</sup> Recalling that the immediate background for this theorem was Bradwardine's *De proportionibus velocitatum in motibus* (1328), and that it was formulated in the context of the undergraduate disputations on *sophismata*, she explains that the key-concept of latitude of velocity might not have been formulated if velocities had continued to be conceived as forms naturally inhering to bodies. Moreover, what the Oxford Calculators developed in this context illustrates the possibility of a science of motion organized as a deductive system on the model of geometry, since consequences are inferred from the initial definitions of uniform and difform motions. Sylla ends up with the question of how one should characterize the Calculators' move: according to her, their philosophical commitment to the ontological minimalism previously promoted by Ockham was an important factor. Ontological nominalism encouraged them to develop a concept, like the concept of latitude of velocity, that does not refer directly to individual things in the outside world. This essay indirectly leads to a question: to what extent should a concept like the latitude of velocity be considered mathematical or to what extent should an inference such as "if the motion is uniformly difform, then the distances traversed in successive units of time will be as the odd numbers" be categorized as mathematical rather than as logical? Mathematical disciplines may articulate rules of logical inference, put constraints on manipulation of formalisms, develop graphic notations, symbolic practices, techniques for solving problems, etc. From a broader perspective, one could argue that the Calculators explored one of the many facets of the

---

<sup>22</sup> Here I can not help referring to George-Théodule Guilbaud, *Mathématique sociale*, entretien avec E. Coumet, P. de Mendez et P. Rosenstiehl, *Savoir et Mémoire*, 4 (Paris, 1993), 3 : "[...] la mathématique, c'est un exercice : on ne "sait" pas des mathématiques, on en "fait"! [...] S'il n'y avait pas de mathématiciens, est-ce qu'il y aurait, dans l'histoire de l'humanité, de la mathématique ? Voilà le problème, c'est l'absence d'objet véritable, d'où la nécessité de les fabriquer et de les refabriquer [...]. [O]n n'applique pas les mathématiques à quelque chose. Disons plutôt qu'il y a un réel, un donné, et si j'entre dans cet univers-là avec un esprit mathématisant, je vois des choses que peut-être d'autres n'auraient pas vues".

<sup>23</sup> This essay follows on previous papers written by Sylla. Among the more recent ones, see "Mathematical Physics and Imagination in the Work of the Oxford Calculators: Roger Swineshead's *On Natural Motions*," in *Mathematics and Its Application to Science and Natural Philosophy in the Middle Ages*, ed. Edward Grant and John Murdoch (Cambridge, 1987), 69–101; "The Oxford Calculators and Mathematical Physics: John Dumbleton's *Summa Logicae et Philosophiae Naturalis*, Parts II and III," in *Physics, Cosmology and Astronomy, 1300-1700: Tension and Accommodation*, ed. Sabetai Unguru (Dordrecht, 1991), 129–61; "Thomas Bradwardine's *De continuo* and the Structure of Fourteenth-Century Learning," in *Texts and Contexts in Ancient and Medieval Science. Studies on the Occasion of John E. Murdoch's Seventieth Birthday*, ed. Edith Sylla and Michael McVaugh (Leiden, 1997); "The Origin and Fate of Thomas Bradwardine's *De proportionibus velocitatum in motibus* in Relation to the History of Mathematics," in *Mechanics and Natural Philosophy*, 67–119.

application of mathematics to motion that later appeared more fully realized in the Galilean science of motion.

The ontological disengagement that, according to Sylla, Oxford's Calculators practiced, appears as well in Boulier's essay.<sup>24</sup> Boulier offers a synthetic account of a well-known paradox: while Galileo believed that the continuum is actually composed of indivisibles, he rejected, officially at least, Cavalieri's method of the indivisibles. Boulier starts from the opposition between Galileo's and Cavalieri's views concerning indivisibles. For Galileo, indivisibles are physical realities, the properties of which can be studied (even if they lead to paradoxes); while for Cavalieri, indivisibles are embodied in a method relying on the equivalence between sets of infinite indivisibles and finite magnitudes. To make this contrast more precise, Boulier proceeds in three steps. First, he studies the paradoxes concerning indivisibles presented in the First Day of the *Discorsi*; second, he examines Cavalieri's method of indivisibles, which remains neutral with regards to their existence as ultimate components of a plane figure; last, he argues that, in the Third Day of the *Discorsi*, Galileo does not rely on the Cavalierian method because he would have remained attached to the "ideal of intelligibility" expressed in the theory of proportions. Perhaps the theory of proportions should be seen less as an abstract ideal of intelligibility, than as a set of rules and general procedures that constrains mathematical reasoning and help it become organized in a coherent whole. On the other hand, in the method of indivisibles, in as far as there is no procedure for identifying "all the lines" of a plane figure, they are not identified except by being seen on the diagram. If this is true, the reason why Galileo does not rely on the method of indivisibles would be that this method was not associated with a procedure sufficiently independent of the diagrams.

In Palmerino's essay, on the other hand, the function of geometrical diagrams in Galileo's geometrization of motion comes under scrutiny. It has been argued that diagrams, being associated with the specific form of mathematization that geometrization is, were doomed to disappear once the abstract procedures of the infinitesimal and differential calculus were applied to the analysis of motion: by the end of the seventeenth century, the mathematics of motion did not proceed by drawing pictures, but by writing equations.<sup>25</sup> Palmerino focuses, however, on precisely the period when diagrams were crucial to the mathematization of motion in order to define their exact function in an historiographical context distinguished by a renewed interest in diagrams.<sup>26</sup> According to her, the unpublished diagrams of acceleration that Galileo drew helped him to understand that the degree of

---

<sup>24</sup> Preliminary studies to Boulier's essay are Enrico Giusti, *Bonaventura Cavalieri and the Theory of Indivisibles* (Rome, 1980); Kirsti Andersen, "Cavalieri's Method of Indivisibles," *Archive for History of Exact Sciences*, 31 (1985), 291-367; Giusti, "Galilei e le leggi"; Michel Blay and Egidio Festa, "Mouvement, continu et composition des vitesses au XVII<sup>e</sup> siècle," *Archives internationales d'histoire des sciences*, 48 (1998), 65-118; Carla Rita Palmerino, "Una nuova scienza della materia per la *scienza nova* del moto. La discussione dei paradossi dell'infinito nella prima giornata dei *Discorsi galileiani*," in *Atomismo e continuo nel XVII secolo. Atti del Convegno Internazionale (Napoli, 28-30 aprile 1997)*, ed. Egidio Festa and Romano Gatto (Naples, 2000), 275-319.

<sup>25</sup> Michael Mahoney, "Diagrams and Dynamics: Mathematical Perspectives on Edgerton's Thesis," in *Science and the Arts*, ed. J. W. Shirley and F. D. Hoeniger (Cranbury, NJ, 1985), 198-220; idem, "Drawing Mechanics," in *Picturing Machines 1400-1700*, ed. Wolfgang Lefèvre (Cambridge, 2004), 281-306. The de-geometrization is obviously not specific to mechanics, but should be rather seen as the disentanglement of analysis itself from its first geometrical context.

<sup>26</sup> See for example Judith V. Field, "Renaissance Mathematics: Diagrams for Geometry, Astronomy and Music," *Interdisciplinary Science Reviews*, 29 (2004), 259-77; Sven Dupré, "Vizualisation in Renaissance Optics: The Function of Geometrical Diagrams and Pictures in the Transmission of Practical Knowledge," in *Transmitting Knowledge: Words, Images, and Instruments in Early Modern Europe*, ed. Sachiko Kusukawa and Ian Maclean (Oxford, 2006), 11-39. On the emergence of the lettered diagram in Greek geometry, see Netz, *The Shaping of Deduction*, 13-88; for a cognitive perspective on diagrams, see, among many studies, Jill H. Larkin and Herbert A. Simon, "Why a Diagram is (Sometimes) Worth Ten Thousand Words," *Cognitive Science*, 11 (1987), 65-99.

velocity in accelerated motion is proportional to the time elapsed and not to the space traversed. In fact, in one of Galileo's diagrams, the space traversed by the falling body is represented at the same time by a line and an area, while in another, the same segment represents both the space traversed and the time elapsed during the fall. Hence, Palmerino's argument is that diagrams were not only material supports of proofs as is the case generally in geometry, but also heuristic tools for the science of nature. According to her, Galileo tacitly assumed that any law of nature could be associated to a coherent geometrical representation. Moreover, she suggests that the reason why Cavalieri and Gassendi, although they endorsed Galileo's law of free fall, replaced his diagram of acceleration with alternative diagrams, may have been that a vertical line figuring in the Galilean diagram could be mistaken for the distance traversed during the fall.

If a moral must be drawn from these first three essays, then surely, it is that the classical mathematization of motion evolves from an interplay of different forms of mathematization, "forms" referring in the present cases to the reasoning of the Calculators, the method of Cavalieri and the diagrams of Galileo. The second group of essays, by Husson, Raynaud and Andraut, deals with case studies devoted to the so-called "application" of mathematics to various practices — from medieval music to early modern anatomy, passing through Renaissance perspective. The common denominator of these three essays is that they examine how mathematics is introduced into practices that may appear more distant from mathematics than mechanics is; namely, music, perspective and anatomy. They differ, however, since they do not expose the introduction of mathematics to the same line of inquiry: Husson studies the theoretical justifications that are given of mathematization, Raynaud analyses its demonstrative effectiveness; Andraut details different aspects of the *more geometrico*.

Husson focuses on the uses of mathematics in the early fourteenth century theory of music; a challenging period since it witnessed the development of polyphonic practices and the transformation of music from a discipline of the *quadrivium* into a *scientia media*.<sup>27</sup> In his introduction, Husson distinguishes two types of mathematization, depending on whether the "descriptive function" of mathematics is extended (this is the case when mathematics is applied to new objects) or the "argumentative function" of mathematics is extended (this is the case when a new argumentative role is ascribed to mathematics). An example of the first type of mathematization is given by Jean de Murs, master of arts at the University of Paris, who used integers to measure not only the pitch, as was usual in his time, but also the duration of a musical sound. Husson shows that de Murs' justification relies on a parallel between sound and motion, which allows one to resort to arguments developed in Aristotle's *Physics*. An example of the second type of mathematization is given by Johannes Boen, a Dutch priest who was responsible for the education of cantors: he used mathematical reasoning to argue that the construction of half-tone is possible, something that would not be the case if music was strictly subordinated to arithmetic. Mathematization of the musical practices thus relies on the interplay between various discourses, from Aristotle's *Physics* to Boethius' *De institutione musica*, passing by considerations on the perfection of the number three.

While Husson examines the justifications given in favor of the introduction of mathematical elements in a practice, Raynaud shows that a practice is sometimes retroactively subjected to hypotheses involving mathematical reasoning. The history of perspective during the Renaissance exhibits what Raynaud felicitously calls a case of "retroactive mathematization." In order to defend a standardized system of perspective

---

<sup>27</sup> Classical studies concerning music in the fourteenth century are Dorit E. Tanay, *Music in the Age of Ockham: The Interrelations between Music, Mathematics, and Philosophy in the 14th Century* (Berkeley, 1989); Frank Hentschel, *Sinnlichkeit und Vernunft in der mittelalterlichen Musiktheorie* (Stuttgart, 2000); Gilles Rico, *Music in the Arts Faculty of Paris in the Thirteenth and Early Fourteenth Centuries* (D. Phil. Thesis, Oxford, 2005).

against those who rejected it, for example, in the name of painters' rules of thumb, mathematical hypotheses that could retroactively explain the choice of this standardized system were sought. The identification of these hypotheses by Piero della Francesca, Giacomo Barozzi da Vignola, Pietro Cataneo or Egnatio Danti, to name just a few, included an arbitrary restriction on the field of vision to a 60° angle, the false hypothesis that the eye does not move, the tentative rejection of curvilinear perspective and the erroneous assumption that natural vision is monocular. As arbitrary or even false as they were from a physiological perspective, these hypotheses were, however, chosen because they could serve as starting points of mathematical proofs and allow for a standardization of practice, in particular in the context of teaching. Hence, the mathematization of perspective implied neither quantification nor axiomatisation, but, rather, relied on the demonstrative power of mathematics and on its potential for standardization and simplification. In his conclusion, Raynaud insists that Renaissance theoreticians of perspective had a dual competence (as mathematicians and as painters, engineers or architects) and that geometrical optics played a mediatory function between geometry and perspective.

In contrast to Raynaud, Andrault focuses on a single book; namely, Niels Steno's *Elementorum Myologiae Specimen* (1667), which is regularly taken as an exemplary case of mathematization since it pretends to geometrize the muscles and their movement of contraction. However, argues Andrault, the *more geometrico* order asserted by Steno has rarely been studied for its own sake.<sup>28</sup> She argues that Steno's aim was neither to offer an ontological reduction of the muscles to mathematical figures nor to give a mathematical explanation of the functioning of the muscles, but, rather, to give a simplified description of the muscular fibers as parallelepipeds and to suggest that their contraction may possibly be explained without making the usual assumption that they swell because of an augmentation of their volume due to the influx of animal spirits. In this case, mathematization is primarily linked to a process of abstraction that amounts to representing the muscular fibers as parallelepipeds. This abstract representation allows a comparison of explanations that does not prejudge which of these explanations happens to be empirically true. Finally, Andrault insists that Steno's *Specimen*, in contrast to, for example, Alfonso Borelli's *De motu animalium* but also contrary to Steno's own anatomical writings, follows a true synthetic order, where only the hypotheses necessary to the subsequent demonstrations are displayed. Hence, Steno offers a rather pure example of the application of elementar geometry to another field of knowledge; as he wrote at the end of the seventeenth century, he did not feel the need of providing long justifications for what he was doing.

The question of mathematization can finally be, so to speak, turned on itself, i.e., related to pure mathematics. In the essay that concludes this fascicle, Maronne shows that, given the variety of mathematics, one can speak without pun of a mathematization of mathematics: applying mathematics to a non-mathematical discipline displays only a difference of degree in comparison to applying a part of mathematics to another part of mathematics. As has been pointed out from the start, mathematics provides symbolic tools that are primarily designed to perform a specific task, even if their application to new purposes may imply that they are slightly modified. Given this, it's only natural that the Cartesian algebraization of geometrical problems didn't turn out to be a transposition that carried no cost.<sup>29</sup> The cost in this case, which, as Maronne recalls, was totally assumed by Descartes

---

<sup>28</sup> Recent studies on Steno include Troels Kardel, "Elements of Myology in Historical Perspective", in *Steno on Muscles*, (Philadelphia, 1994); idem, "Nicolas Steno's New Myology," *Nuncius*, 23 (2008), 37-64; Domenico Bertoloni Meli, "The Collaboration between Anatomists and Mathematicians in the mid-Seventeenth Century with a Study of Images as Experiments and Galileo's Role in Steno's Myology," *Early Science and Medicine*, 13 (2008), 663-709.

<sup>29</sup> The background to Maronne's essay is Henk Bos, *Refracting Geometrical Exactness. Descartes'*

himself, is that it can be difficult to give the actual construction of the geometric objects that correspond to the algebraic solution of the problem proposed. Thus, the Cartesian algebraization of geometry created a new epistemological situation: in certain cases, there might be a discrepancy between the easy algebraic solution of an equation and the difficult geometrical construction of this solution. Referring to classical problems like Apollonius' problem of the three circles or Pappus' problem, but also to the correspondence between Blaise Pascal and René-François de Sluse during the year 1657, Maronne consequently examines the different attitudes mathematicians adopted towards the geometrical construction of algebraic solutions. Descartes thought that the game is not worth the candle. Pascal had proposed a wholly geometrical solution of Pappus's problem that implies an actual construction of all possible cases. Sluse, at last, embodied an intermediary position: on the one hand, he defended Descartes' analysis, while on the other, he was inclined to think that a geometrical construction of the algebraic solution of problem is necessary, even if he did not always provide it.